

# 习 题 答 案

## 第 1 章

### 习题 1.2

1.2-1.  $\tau = 0.064 \text{ s}$

1.2-2.  $f = 5.67 \text{ cps}$

1.2-3.  $\tau = 0.533 \text{ s}$

1.2-4.  $\tau = 1.99 \text{ s}$

1.2-5.  $\sigma_{\max} = 7230 \text{ psi}$

1.2-6.  $\tau_a = 0.813 \text{ s}; \tau_b = 1.62 \text{ s}$

1.2-7.  $f = 5.88 \text{ cps}$

1.2-8.  $\tau = 2\pi \sqrt{ml/(2S)}$

1.2-9.  $k = k_1 k_2 k_3 / (k_1 k_2 + k_1 k_3 + k_2 k_3)$

### 习题 1.3

1.3-1.  $f = 0.935 \text{ cps}$

1.3-2.  $I_2 = I_1(\tau_2^2 - \tau_0^2) / (\tau_1^2 - \tau_0^2)$

1.3-3.  $\tau = 2\pi \sqrt{W/(3kg)}$

1.3-4.  $\tau = 2\pi(l/a) \sqrt{W/(3kg)}$

1.3-5.  $f = \frac{1}{2\pi} \sqrt{\frac{\pi d^4 G (l_1 + l_2)}{32 I l_1 l_2}}$

1.3-6.  $L_1 = 2a + \frac{C_1}{C_2} b + 2 \frac{C_1}{B} r$

1.3-7.  $\tau = 2\pi \sqrt{\frac{Wr(r - r_0)}{ngS_0 r_0}}$

### 习题 1.4

1.4-1.  $\omega = \sqrt{\frac{g}{l}} \sin \beta$

1.4-2.  $f = 2.72 \text{ cps}$

1.4-3.  $W_3 = 27 \text{ N}$

1.4-4.  $\omega = \sqrt{\frac{g}{l} \left[ \frac{3ka^2/l}{3W + wl} - \frac{3}{4} \left( \frac{4W + 2wl}{3W + wl} \right) \right]}$

1.4-5.  $\omega = \sqrt{\frac{g[k_1 a^2 + k_2 (a \tan \alpha)^2]}{Wl^2}}$

1.4-6.  $\omega = \sqrt{\frac{3ga^2}{lb^2}}$

$$\mathbf{1.4-7.} \omega = \sqrt{\frac{cg}{i^2 + (r - c)^2}}$$

## 习题 1.5

$$\mathbf{1.5-1.} \tau = \frac{2\pi}{3.58} \sqrt{\frac{\omega l^4}{EIg}}$$

$$\mathbf{1.5-2.} 13/35$$

$$\mathbf{1.5-3.} \tau = \frac{2\pi}{22.6} \sqrt{\frac{\omega l^4}{EIg}}$$

$$\mathbf{1.5-4.} \tau = 1.73 \text{ s}$$

$$\mathbf{1.5-5.} 239/3360 \approx 1/14$$

$$\mathbf{1.5-6.} \omega = 1.58 \sqrt{\frac{kg}{wl}}$$

$$\mathbf{1.5-7.} \frac{w_1 l_1 k_2^2 + 3w_2 l_2 k_2 (k_1 + k_2) + w_2 l_2 k_1^2}{3(k_1 + k_2)^2}$$

$$\mathbf{1.5-8.} \frac{i_1 l_1 k_{r2}^2 + 3il_1 k_{rl} (k_{rl} + k_{r2}) + i_2 l_2 k_{rl}^2}{3(k_{rl} + k_{r2})^2}$$

$$\mathbf{1.5-9.} f = \frac{1}{2\pi} \sqrt{\frac{ngS_0 r_0}{(W + W_s n/3)r(r - r_0)}}$$

$$\mathbf{1.5-10.} f = \frac{1}{2\pi} \sqrt{\frac{16gB}{[W + (116/105)\omega r]r^3}}$$

$$\mathbf{1.5-11.} \tau = 0.637 \sqrt{\frac{\omega l^4}{EIg}}$$

$$\mathbf{1.5-12.} \tau = 1.84 \sqrt{\frac{\omega l^4}{EIg}}$$

$$\mathbf{1.5-13.} \omega = 3.07 \sqrt{\frac{GJ}{il^2}}$$

$$\mathbf{1.5-14.} \omega = 1.57 \sqrt{\frac{EAg}{\omega l^2}}$$

## 习题 1.6

$$\mathbf{1.6-1.} 0.004 \text{ in.}$$

$$\mathbf{1.6-2.} 3.3 \text{ mm}$$

$$\mathbf{1.6-3.} 0.0078 \text{ in.}$$

$$\mathbf{1.6-4.} 5.1 \text{ mm}$$

$$\mathbf{1.6-5.} 0.0454 \text{ in.}$$

$$\mathbf{1.6-6.} 0.0056 \text{ in.}$$

## 习题 1.7

$$\mathbf{1.7-1.} u_1 = 0.147 \text{ in.}$$

$$\dot{u}_1 = -1.22 \text{ in./s}$$

**1.7-2.**  $u_1 = -0.035 \text{ in.}$

$$\dot{u}_1 = -1.81 \text{ in./s}$$

**1.7-3.**  $u = \frac{qt}{2\omega} \sin \omega t$

### 习题 1.8

**1.8-1.**  $0.539/1$

**1.8-2.**  $\dot{u}_{\max} = -0.24 \text{ m/s}$

**1.8-3.**  $t = 0.50 \text{ s}$

$$-u_{\max} = 0.000496 \text{ in.}$$

**1.8-4.**  $c = 0.00143 \text{ lb-s/in.}$

**1.8-5.**  $f_d = \sqrt{3/2} f$

### 习题 1.9

**1.9-1.**  $A = 0.803 \text{ in.}$

**1.9-2.**  $A = 2.85 \text{ mm}$

**1.9-3.**  $\beta_{\max} = \frac{1}{\sqrt{1 - \Omega^4/\omega^4}}$

**1.9-4.**  $u = \frac{Q}{k} \beta \sin(\Omega t - \theta)$

$$\theta = \arctan\left(\frac{2n\Omega}{\omega^2 - \Omega^2}\right)$$

**1.9-5.**  $u = \frac{Bd}{k} \beta \sin(\Omega t - \phi - \theta)$

$$\phi = \arctan\left(\frac{c\Omega}{-k}\right)$$

**1.9-6.**  $u^* = \frac{-ma}{k} \beta \cos(\Omega t - \theta)$

$$\theta = \arctan\left(\frac{2n\Omega}{\omega^2 - \Omega^2}\right)$$

**1.9-7.**  $u^* = \frac{-ma}{k} \beta \sin(\Omega t - \theta)$

$$\theta = \arctan\left(\frac{2n\Omega}{\omega^2 - \Omega^2}\right)$$

**1.9-8.**  $u_{tr} = -e^{-nt} \left( -N \cos \omega_d t + \frac{M\Omega - Nn}{\omega_d} \sin \omega_d t \right)$

### 习题 1.11

**1.11-2.**  $F(t) = \frac{4P}{\pi} \left( \sin \Omega t + \frac{1}{3} \sin 3\Omega t + \dots \right)$

$$1.11-3. F(t) = \frac{4P}{\pi} \left( \cos \Omega t - \frac{1}{3} \cos 3\Omega t + \dots \right)$$

$$1.11-4. F(t) = \frac{2P}{\pi} \left( \sin \Omega t - \frac{1}{2} \sin 2\Omega t + \frac{1}{3} \sin 3\Omega t - \dots \right)$$

$$1.11-5. F(t) = \frac{P}{2} - \frac{P}{\pi} \left( \sin \Omega t + \frac{1}{2} \sin 2\Omega t + \frac{1}{3} \sin 3\Omega t + \dots \right)$$

$$1.11-6. u = \frac{a_0}{k} + \sum_{i=1}^{\infty} \frac{a_i \cos(i\Omega t - \theta_i) + b_i \sin(i\Omega t - \theta_i)}{k \sqrt{(1 - i^2 \Omega^2 / \omega^2)^2 + (2\gamma i \Omega / \omega)^2}}$$

### 习题 1.12

$$1.12-1. u = \frac{Q_1}{k} (1 - \cos \omega t) \quad (0 \leq t \leq t_1)$$

$$u = \frac{Q_1}{k} [\cos \omega(t - t_1) - \cos \omega t] - \frac{Q_2}{k} [1 - \cos \omega(t - t_1)] \quad (t_1 \leq t \leq t_2)$$

$$u = \frac{Q_1}{k} [\cos \omega(t - t_1) - \cos \omega t] - \frac{Q_2}{k} [\cos \omega(t - t_2) - \cos \omega(t - t_1)] \quad (t_2 \leq t)$$

$$1.12-2. u = \frac{Q_1}{k} \left( 1 - \cos \omega t - \frac{t}{t_1} + \frac{\sin \omega t}{\omega t_1} \right) \quad (0 \leq t \leq t_1)$$

$$u = \frac{Q_1}{k} \left[ -\cos \omega t + \frac{\sin \omega t - \sin \omega(t - t_1)}{\omega t_1} \right] \quad (t_1 \leq t)$$

$$1.12-3. u = \frac{Q_1}{k} (1 - \cos \omega t) \quad (0 \leq t \leq t_1)$$

$$u = \frac{Q_1}{k} \left[ 1 - \cos \omega t - \frac{t - t_1}{t_2 - t_1} + \frac{\sin \omega(t - t_1)}{\omega(t_2 - t_1)} \right] \quad (t_1 \leq t \leq t_2)$$

$$u = \frac{Q_1}{k} \left[ -\cos \omega t + \frac{\sin \omega(t - t_1) - \sin \omega(t - t_2)}{\omega(t_2 - t_1)} \right] \quad (t_2 \leq t)$$

$$1.12-4. u = \frac{Q_1}{k} \left( \frac{t}{t_1} - \frac{\sin \omega t}{\omega t_1} \right) \quad (0 \leq t \leq t_1)$$

$$u = \frac{Q_1}{k} \left[ \frac{t}{t_1} - \frac{\sin \omega t}{\omega t_1} - \frac{t_2(t - t_1)}{t_1(t_2 - t_1)} + \frac{t_2 \sin \omega(t - t_1)}{\omega t_1(t_2 - t_1)} \right] \quad (t_1 \leq t \leq t_2)$$

$$u = \frac{Q_1}{k} \left[ -\frac{\sin \omega t}{\omega t_1} + \frac{t_2 \sin \omega(t - t_1)}{\omega t_1(t_2 - t_1)} - \frac{\sin \omega(t - t_2)}{\omega(t_2 - t_1)} \right] \quad (t_2 \leq t)$$

$$1.12-5. u = \frac{Q_1}{k} \left[ \left( 1 + \frac{2}{\omega^2 t_1^2} \right) (1 - \cos \omega t) - \frac{t^2}{t_1^2} \right] \quad (0 \leq t \leq t_1)$$

$$u = \frac{Q_1}{k} \left\{ \frac{2}{\omega^2 t_1^2} [\cos \omega(t - t_1) - \cos \omega t] - \frac{2}{\omega t_1} \sin \omega(t - t_1) - \cos \omega t \right\} \quad (t_1 \leq t)$$

$$1.12-6. u = \frac{Q_1}{k} \left[ \left( 1 - \frac{2}{\omega^2 t_1^2} \right) (1 - \cos \omega t) - \frac{2t}{t_1} + \frac{t^2}{t_1^2} + \frac{2 \sin \omega t}{\omega t_1} \right] \quad (0 \leq t \leq t_1)$$

$$u = \frac{Q_1}{k} \left\{ \frac{2}{\omega^2 t_1^2} [\cos \omega t - \cos \omega(t - t_1)] - \cos \omega t + \frac{2 \sin \omega t}{\omega t_1} \right\} \quad (t_1 \leq t)$$

$$1.12-7. u = \frac{\delta Q}{k} \left[ t - \frac{2n}{\omega^2} + e^{-mt} \left( \frac{2n}{\omega^2} \cos \omega_d t - \frac{\omega_d^2 - n^2}{\omega^2 \omega_d} \sin \omega_d t \right) \right]$$

**习题 1.13**

**1.13-1.**  $u = d \left( \frac{t}{t_1} - \frac{\sin \omega t}{\omega t_1} \right) \quad (0 \leq t \leq t_1)$

$$u = d \left[ \frac{\sin \omega(t-t_1) - \sin \omega t}{\omega t_1} + \cos \omega(t-t_1) \right] \quad (t_1 \leq t)$$

**1.13-2.**  $u^* = -\frac{a}{\omega^2} \left( \frac{t}{t_1} - \frac{\sin \omega t}{\omega t_1} \right) \quad (0 \leq t \leq t_1)$

$$u^* = -\frac{a}{\omega^2} \left[ 1 + \frac{\sin \omega(t-t_1) - \sin \omega t}{\omega t_1} \right] \quad (t_1 \leq t)$$

**1.13-3.**  $u = d_1(1 - \cos \omega t) - (d_1 + d_2) \left( \frac{t}{t_1} - \frac{\sin \omega t}{\omega t_1} \right) \quad (0 \leq t \leq t_1)$

$$u = -d_1 \cos \omega t - d_2 \cos \omega(t-t_1) - \frac{d_1 + d_2}{\omega t_1} [\sin \omega(t-t_1) - \sin \omega t] \quad (t_1 \leq t)$$

**1.13-4.**  $u^* = -\frac{a_1}{\omega^2}(1 - \cos \omega t) - \frac{a_2 - a_1}{\omega^2} \left( \frac{t}{t_1} - \frac{\sin \omega t}{\omega t_1} \right) \quad (0 \leq t \leq t_1)$

$$u^* = -\frac{a_1}{\omega^2} [\cos \omega(t-t_1) - \cos \omega t]$$

$$-\frac{a_2 - a_1}{\omega^2} \left[ \frac{\sin \omega(t-t_1) - \sin \omega t}{\omega t_1} + \cos \omega(t-t_1) \right] \quad (t_1 \leq t)$$

**1.13-5.**  $u = d \left[ \frac{2}{\omega^2 t_1^2} (1 - \cos \omega t - \omega t_1 \sin \omega t) + \frac{2t}{t_1} - \frac{t^2}{t_1^2} \right] \quad (0 \leq t \leq t_1)$

$$u = d \left\{ \frac{2}{\omega^2 t_1^2} [\cos \omega(t-t_1) - \cos \omega t - \omega t_1 \sin \omega t + \cos \omega(t-t_1)] \right\} \quad (t_1 \leq t)$$

**1.13-6.**  $u^* = -\frac{a}{\omega^2} \left[ \frac{t^2}{t_1^2} - \frac{2}{\omega^2 t_1^2} (1 - \cos \omega t) \right] \quad (0 \leq t \leq t_1)$

$$u^* = -\frac{a}{\omega^2} \left\{ \frac{2}{\omega^2 t_1^2} [\cos \omega t - \cos \omega(t-t_1) + \omega t_1 \sin \omega(t-t_1)] + \cos \omega(t-t_1) \right\} \quad (t_1 \leq t)$$

**1.13-7.**  $A^* = \frac{\sqrt{2a}}{\omega^2} \sqrt{1 - \cos \left( \frac{\omega u_0}{a} \right)}$

**习题 1.15**

**1.15-1.**  $u_{10} = 0$

**1.15-2.**  $u_{10} = \frac{Q_1}{k}$

**1.15-3.** 表 1.1 已给出

**1.15-4.** 表 1.2 已给出

**1.15-5.**  $u_{20} = -2.546 \frac{Q_1}{k}$

**1.15-6.**  $u_{20} = -4 \frac{Q_1}{k}$

**1.15-7.**  $u_{10} = 0.9135 \frac{Q_1}{k}$

$$\mathbf{1.15-8.} u_{10} = 0.3186 \frac{Q_1}{k}$$

$$\mathbf{1.15-9.} u_{10} = 1.379 \frac{Q_1}{k}$$

$$\mathbf{1.15-10.} u_{10} = 1.102 \frac{Q_1}{k}$$

## 第 2 章

### 习题 2.1

$$\mathbf{2.1-1.} (a) m\ddot{u} + 2ku(2 - l/\sqrt{l^2 + u^2}) = 0$$

$$(b) m\ddot{u} + 2ku(1 + u^2/2l^2) = 0$$

$$(c) m\ddot{u} + 2ku = 0$$

$$\mathbf{2.1-2.} (a) m\ddot{u} + 2k \left\{ 2u - l \left[ \frac{1}{\sqrt{1+1/(1+\sqrt{2}u/l)^2}} - \frac{1}{\sqrt{1+1/(1-\sqrt{2}u/l)^2}} \right] \right\} = 0$$

$$(b) m\ddot{u} + 2ku = 0$$

$$\mathbf{2.1-3.} (a) \ddot{\phi} + \frac{g}{l_1} \left[ \sin \phi + \frac{F}{W} \cos(\phi - \theta) \right] = 0$$

其中

$$F = k \left( \frac{l_2 + l_1 \sin \phi}{\cos \theta} - l_2 \right); \quad \theta = \arctan \left[ \frac{l_1(1 - \cos \phi)}{l_2 + l_1 \sin \phi} \right]$$

$$(b) \ddot{\phi} + \left( \frac{g}{l_1} + \frac{kg}{W} \right) \phi = 0$$

$$\mathbf{2.1-4.} (a) \ddot{\phi} + \frac{g}{l_1} \sin \phi + \frac{kg}{Wl_1} (l_1 + l_2) \times \left[ 1 - \frac{l_2}{\sqrt{l_2^2 + 2l_1(l_1 + l_2)(1 - \cos \phi)}} \right] \sin \phi = 0$$

$$(b) \ddot{\phi} + \frac{g}{l_1} \phi + \frac{g}{2} \left[ \frac{k}{W} \left( \frac{l_1}{l_2} + 1 \right)^2 - \frac{1}{3l_1} \right] \phi^3 = 0$$

$$(c) \ddot{\phi} + \frac{g}{l_1} \phi = 0$$

$$\mathbf{2.1-5.} m\ddot{u} + k \left[ u - l \cos \theta + \frac{l(l \cos \theta - u)}{\sqrt{l^2 + u^2 - 2lu \cos \theta}} \right] = 0$$

$$\mathbf{2.1-6.} \ddot{\phi} + \frac{k_r g}{Wl^2} \phi - \frac{g}{l} \sin \phi = 0$$

### 习题 2.2

$$\mathbf{2.2-1.} u_m = 2.13 \text{ in.}; P_m = 8563 \text{ lb}; t_m = \tau/4 = 0.308 \text{ s}$$

$$\mathbf{2.2-2.} u_m = 4.75 \text{ in.}; P_m = 2350 \text{ lb}$$

$$\mathbf{2.2-3.} u_m = 3.16 \text{ in.}; P_m = 3170 \text{ lb}$$

$$\mathbf{2.2-4.} u_m = 3.18 \text{ in.}; P_m = 3130 \text{ lb}$$

### 习题 2.5

$$\mathbf{2.5-1.} t_n = t_1 + \frac{1}{\omega} \arctan [(Q_n/k)(\dot{u}_1/\omega)]$$

$$A = \sqrt{\left(u_1 + \frac{Q_n}{k}\right)^2 + \left(\frac{Q_n}{k}\right)^2 + \left(\frac{\dot{u}_1}{\omega}\right)^2}$$

$$2.5-2. t_m = \frac{\pi}{2} \left( \sqrt{\frac{m}{k_1}} + \sqrt{\frac{m}{k_2}} \right); u_m = u_1 \left[ 1 + \sqrt{\frac{k_1}{k_2}} \right]$$

$$2.5-3. t_n = \frac{\pi}{2} \sqrt{\frac{m}{k_1}}; A = u_1 \left[ 1 + \frac{k_1}{k_2} \left( 1 - \sqrt{1 + \frac{k_1}{k_2}} \right) \right]$$

$$2.5-4. t_m = 3.83 \sqrt{m/k}; u_m = 3u_1/2$$

$$2.5-5. u_m = 3u_1/2; u_r = u_1/2$$

2.5-6. 2.26 s

2.5-7. 0.25

$$2.5-8. \tau = \pi(\sqrt{m/k_1} + \sqrt{m/k_2}); u_m = \dot{u}_0 \sqrt{m/k_2}$$

$$2.5-9. \tau = \frac{2}{\omega} \arctan \left( \frac{k \dot{u}_0}{P_1 \omega} \right); u_m = \sqrt{\left( \frac{\dot{u}_0}{\omega} \right)^2 + \left( \frac{P_1}{k} \right)^2} - \frac{P_1}{k}$$

$$2.5-10. \tau = 4t_1 + \frac{4}{\omega} \arctan \left( \frac{k_2 \dot{u}_1}{k_1 \omega_2 u_1} \right)$$

$$u_m = \left( 1 - \frac{k_1}{k_2} \right) u_1 + \sqrt{\left( \frac{k_1}{k_2} u_1 \right)^2 + \left( \frac{\dot{u}_1}{\omega_2} \right)^2}$$

$$2.5-11. u_m = u_1(1 + \sqrt{5}); u_r = 4u_1/\sqrt{5}$$

$$2.5-12. u_m = u_1(6 + 2\sqrt{10}); u_r = 4u_1 \left[ 1 + 2\sqrt{\frac{2}{5}} \right]$$

## 习题 2.6

2.6-1. 表 2.1 已给出

2.6-2. 表 2.1 已给出

2.6-3.  $\phi_{10} \approx \pi/2$  rad

2.6-4.  $\dot{u}_{\min} \approx -1.59$  in./s

2.6-5.  $\phi_{\max} \approx \pi/2$  rad

2.6-6.  $u_m \approx 4.75$  in.;  $t_m \approx 0.725$  s

2.6-7.  $u_m \approx 3.18$  in.;  $t_m \approx 0.50$  s

2.6-8.  $u_m \approx 1.50$  in.;  $t_m \approx 0.75$  s

2.6-9.  $u_m \approx 1.51$  in.;  $t_m \approx 1.20$  s

2.6-10.  $u_m \approx 3.24$  in.;  $t_m \approx 1.60$  s

## 第 3 章

### 习题 3.5

3.5-1.  $r_1 = 1; r_2 = -1$

3.5-2.  $r_1 = -\sqrt{3}/3; r_2 = \sqrt{3}$

3.5-3.  $r_1 = 1/(1+\sqrt{2}); r_2 = 1/(1-\sqrt{2})$

3.5-4.  $r_1 = 1/\sqrt{2}; r_2 = -1/\sqrt{2}$

3.5-5.  $r_1 = 0.321; r_2 = -3.12$

**3.5-6.**  $r_1 = 2.41$ ;  $r_2 = -0.414$

**3.5-7.**  $r_1 = 1$ ;  $r_2 = -1$

**3.5-8.**  $r_1 = 1/(1+\sqrt{2})$ ;  $r_2 = 1/(1-\sqrt{2})$

**3.5-11.**  $r_1 = -13.4 \text{ ft/rad}$ ;  $r_2 = -2.41 \text{ ft/rad}$

**3.5-12.**  $r_1 = 0.578l$ ;  $r_2 = -0.578l$

## 第 4 章

### 习题 4.2

**4.2-1.**  $\omega_{1,2,3}^2 = (2-\sqrt{2})T/(ml)$ ,  $2T/(ml)$ ,  $(2+\sqrt{2})T/(ml)$

**4.2-2.**  $\lambda_{1,2,3} = (2+\sqrt{2})m/(2k)$ ,  $m/(2k)$ ,  $(2-\sqrt{2})m/(2k)$

**4.2-3.**  $\omega_{1,2,3}^2 = \frac{g}{l}$ ,  $\frac{g}{l} + \frac{kh^2}{ml^2}$ ,  $\frac{g}{l} + \frac{3kh^2}{ml^2}$

**4.2-4.**  $\lambda_{1,2,3} = 5.05I/k_r$ ,  $0.643I/k_r$ ,  $0.308I/k_r$

**4.2-5.**  $\omega_{1,2,3,4}^2 = 0$ ,  $(2-\sqrt{2})k/m$ ,  $2k/m$ ,  $(2+\sqrt{2})k/m$

**4.2-6.**  $\lambda_{1,2,3} = 31.6\alpha$ ,  $2\alpha$ ,  $0.444\alpha$  [ $\alpha = ml^3/(768EI)$ ]

**4.2-7.**  $\omega_{1,2,3}^2 = 0.416g/l$ ,  $2.29g/l$ ,  $6.29g/l$

**4.2-8.**  $\lambda_{1,2,3} = 14.4\alpha$ ,  $2.62\alpha$ ,  $0.954\alpha$  [ $\alpha = ml^3/(144EI)$ ]

**4.2-9.**  $\omega_{1,2,3}^2 = 0.332k/m$ ,  $1.28k/m$ ,  $1.39k/m$

**4.2-10.**  $\lambda_{1,2,3} = 74.4\alpha$ ,  $6.99\alpha$ ,  $0.307\alpha$  [ $\alpha = ml^3/(48EI)$ ]

**4.2-11.**  $\omega_{1,2,3}^2 = 0$ ,  $0$ ,  $9EI/ml^3$

**4.2-12.**  $\omega_{1,2,3}^2 = (3-\sqrt{3})k/m$ ,  $3k/m$ ,  $(3+\sqrt{3})k/m$

### 习题 4.4

**4.4-1.**  $u_1 = P(1.220\cos\omega_1 t - 0.280\cos\omega_2 t + 0.060\cos\omega_3 t)/k$

**4.4-2.**  $u_1 = \dot{u}_{01}(\sin\omega_2 t)\omega_2$

**4.4-3.**  $\theta_1 = \phi(\cos\omega_1 t - \cos\omega_3 t)/3$

**4.4-4.**  $\phi_1 = \dot{\theta}[0.543(\sin\omega_1 t)/\omega_1 + 0.349(\sin\omega_2 t)/\omega_2 + 0.108(\sin\omega_3 t)/\omega_3]$

**4.4-5.**  $u_1 = \dot{u}_{01}[(t + \sin\omega_3 t)/\omega_3]/2$

**4.4-6.**  $v_1 = \dot{\theta}l[1.707(\sin\omega_1 t)/\omega_1 - (\sin\omega_2 t)/\omega_2 + 0.293(\sin\omega_3 t)/\omega_3]/4$

**4.4-7.**  $u_1 = \Delta(0.334\cos\omega_1 t + 0.314\cos\omega_2 t + 0.352\cos\omega_3 t)$

**4.4-8.**  $u_1 = Ph^3(2.616\cos\omega_1 t - 0.702\cos\omega_2 t + 0.083\cos\omega_3 t)/(144EI)$

**4.4-9.**  $u_1 = \dot{u}_{01}[0.708(\sin\omega_1 t)/\omega_1 + 0.292(\sin\omega_3 t)/\omega_3]$

**4.4-10.**  $u_1 = Pl^3(26.33\cos\omega_1 t - 2.315\cos\omega_2 t - 0.007\cos\omega_3 t)/(48EI)$

**4.4-11.**  $v_1 = \dot{v}_{01}[4t - (\sin\omega_3 t)/\omega_3]/3$

**4.4-12.**  $v_1 = \Delta(1.367\cos\omega_1 t - 0.367\cos\omega_3 t)$

### 习题 4.5

**4.5-1.**  $u_1 = (P\cos\Omega t)(0.242\beta_1/\omega_1^2 - 0.436\beta_2/\omega_2^2 + 0.194\beta_3/\omega_3^2)/m$

**4.5-2.**  $u_1 = P[6 - (2+\sqrt{2})\cos\omega_1 t - 2\cos\omega_2 t - (2-\sqrt{2})\cos\omega_3 t]/(8k)$

**4.5-3.**  $\theta_1 = \frac{R}{3ml} \left[ \left( t - \frac{1}{\omega_1} \sin \omega_1 t \right) / \omega_1^2 - \left( t - \frac{1}{\omega_3} \sin \omega_3 t \right) / \omega_3^2 \right]$

**4.5-4.**  $\phi_1 = (T \sin \Omega t) (0.218\beta_1/\omega_1^2 - 0.097\beta_2/\omega_2^2 - 0.121\beta_3/\omega_3^2)/I$

**4.5-5.**  $u_1 = P[t^2 + (1 - \cos \omega_3 t)m/k]/(4m)$

**4.5-6.**  $v_1 = \frac{R}{2m} \left[ \left( t - \frac{1}{\omega_1} \sin \omega_1 t \right) / \omega_1^2 + \left( t - \frac{1}{\omega_3} \sin \omega_3 t \right) / \omega_3^2 \right]$

**4.5-7.**  $u_1 = (P \cos \Omega t) (0.077\beta_1/\omega_1^2 + 0.290\beta_2/\omega_2^2 + 0.132\beta_3/\omega_3^2)/m$

**4.5-8.**  $u_1 = Ph^3 (5.995 - 4.805 \cos \omega_1 t - 0.871 \cos \omega_2 t - 0.319 \cos \omega_3 t)/(144EI)$

**4.5-9.**  $w_1 = \frac{R}{\omega_2^2 m} \left[ \left( t - \frac{1}{\omega_2} \sin \omega_2 t \right) \right]$

**4.5-10.**  $u_1 = (P \sin \Omega t) (0.094\beta_1/\omega_1^2 - 0.145\beta_2/\omega_2^2 + 0.052\beta_3/\omega_3^2)/m$

**4.5-11.**  $v_1 = \frac{R}{9m} \left[ t^3 + 3 \left( t - \frac{1}{\omega_3} \sin \omega_3 t \right) / \omega_3^2 \right]$

**4.5-12.**  $v_1 = P(0.0379 - 0.0647 \cos \omega_1 t + 0.0251 \cos \omega_3 t)/k$

#### 习题 4.6

**4.6-1.**  $u_1 = kd_1 [0.108f_1(t) + 0.543f_2(t) + 0.349f_3(t)]/t_1 m; f_1(t) = \left( t - \frac{1}{\omega_1} \sin \omega_1 t \right) / \omega_1^2$ , 等

**4.6-2.**  $u_1 = d[4 - (2 + \sqrt{2}) \cos \omega_1 t - (2 - \sqrt{2}) \cos \omega_3 t]/4$

**4.6-3.**  $\theta_1 = -(a/g)\beta_1 \sin \Omega t$

**4.6-4.**  $\phi_1^* = -\alpha_1 I [3t^2 - I(28.01 - 27.70 \cos \omega_1 t - 0.289 \cos \omega_2 t - 0.020 \cos \omega_3 t)/k_r]/(t_1^2 k_r)$

**4.6-5.**  $u_1 = kd_1 [0.242f_1(t) - 0.436f_2(t) + 0.194f_3(t)]/(t_1 m);$

$f_1(t) = \left( t - \frac{1}{\omega_1} \sin \omega_1 t \right) / \omega_1^2$ , 等

**4.6-6.**  $v_1 = d(3 - 1.707 \cos \omega_1 t - \cos \omega_2 t - 0.293 \cos \omega_3 t)/4$

**4.6-7.**  $u_1 = d(1 - 0.334 \cos \omega_1 t - 0.314 \cos \omega_2 t - 0.352 \cos \omega_3 t)$

**4.6-8.**  $u_1^* = -a \sin \Omega t (0.333\beta_1/\omega_1^2 + 0.333\beta_2/\omega_2^2 + 0.334\beta_3/\omega_3^2)$

**4.6-9.**  $u_1 = d(1 - 0.708 \cos \omega_1 t - 0.292 \cos \omega_3 t)$

**4.6-10.**  $u_1 = \phi l \cos \Omega t (1.242\beta_1 - 0.249\beta_2 + 0.008\beta_3)$

**4.6-11.**  $v_1^* = v_3^* = -a_1 [t^2 - 2(1 - \cos \omega t) / \omega^2] \omega^2 t_1^2$ , 其中  $\omega^2 = 3EI/l^3 m$

**4.6-12.**  $v_1 = d \sin \Omega t (0.096\beta_1 - 0.096\beta_3)$

#### 习题 4.7

**4.7-1.**  $\lambda_{1,2,3} \approx 2.618ml/T, 0.500ml/T, 0.382ml/T$

**4.7-2.**  $\lambda_{1,2,3} \approx 2.618m\delta, 1.000m\delta, 0.382m\delta$

**4.7-3.**  $\lambda_{1,2,3} \approx 39.68\alpha, 2.815\alpha, 0.501\alpha [\alpha = ml^3/(768EI)]$

**4.7-4.**  $\lambda_{1,2,3} \approx 0.246g/l, 1.252g/l, 2.169g/l$

**4.7-5.**  $\lambda_{1,2,3} \approx 19.12\alpha, 4.000\alpha, 1.884\alpha [\alpha = mh^3/(144EI)]$

**4.7-6.**  $\lambda_{1,2,3} \approx 76.32\alpha, 8.978\alpha, 0.700\alpha [\alpha = ml^3/(48EI)]$

**4.7-7.**  $\omega_{1,2,3}^2 = 0, 0, 6EI/ml^3$

## 第 5 章

### 习题 5.2

$$\mathbf{5.2-1.} u = \sum_{i=1, 3, 5, \dots}^{\infty} \cos \frac{i\pi x}{2l} \left( A_i \cos \frac{i\pi at}{2l} + B_i \sin \frac{i\pi at}{2l} \right)$$

$$\mathbf{5.2-2.} u = \frac{4vl}{\pi^2 a} \sum_{i=1, 3, 5, \dots}^{\infty} \frac{1}{i^2} \cos \frac{i\pi x}{l} \sin \frac{i\pi at}{l}$$

$$\mathbf{5.2-3.} u = \frac{P_0 l}{\pi^2 EA} \sum_{i=2, 6, 10, \dots}^{\infty} \frac{(-1)^{(i-2)/4}}{i^2} \sin \frac{i\pi x}{l} \cos \frac{i\pi at}{l}$$

$$\mathbf{5.2-4.} u = \frac{16P_0 l}{\pi^2 EA} \sum_{i=1, 3, 5, \dots}^{\infty} \frac{1}{i^3} \sin \frac{i\pi x}{2l} \cos \frac{i\pi at}{2l}$$

### 习题 5.3

$$\mathbf{5.3-1.} u = \frac{2lP}{\pi^2 a^2 \rho A} \sum_{i=1, 3, 5, \dots}^{\infty} \frac{(-1)^{(i-1)/2}}{i^2} \sin \frac{i\pi x}{l} \left( 1 - \cos \frac{i\pi at}{2l} \right)$$

$$\mathbf{5.3-2.} u = \frac{P_1 t^3}{6\rho Alt_1} + \frac{2lP_1}{\pi^2 a^2 \rho At_1} \sum_{i=1}^{\infty} \frac{1}{i^2} \cos \frac{i\pi x}{l} \left( t - \frac{l}{i\pi a} \sin \frac{i\pi at}{l} \right)$$

$$\mathbf{5.3-3.} u = \frac{4P_1 \sin \Omega t}{\pi \rho Al} \sum_{i=1, 3, 5, \dots}^{\infty} \frac{\sin(\omega_i x / a)}{i(\omega_i^2 - \Omega^2)}; \quad \omega_i = \frac{i\pi a}{2l}$$

$$\mathbf{5.3-4.} u = \frac{8lP}{\pi^2 a^2 \rho A} \sum_{i=1, 3, 5, \dots}^{\infty} \frac{\cos(i\pi/4)}{i^2} \cos \frac{i\pi x}{2l} \left( 1 - \cos \frac{i\pi at}{2l} \right)$$

### 习题 5.4

$$\mathbf{5.4-1.} u = \frac{2P_0}{3\pi EA} \sum_{i=1}^{\infty} \frac{1}{i} \sin \frac{i\pi x}{l} \left[ \frac{l}{i\pi} \left( 3 \sin \frac{i\pi}{3} - \sin \frac{2i\pi}{3} \right) - \frac{l}{3} \cos \frac{2i\pi}{3} \right] \cos \frac{i\pi at}{l}$$

$$\mathbf{5.4-2.} u = \frac{8vl}{\pi^2 a} \sum_{i=1, 3, 5, \dots}^{\infty} \frac{1}{i^2} \cos \frac{i\pi}{4} \sin \frac{i\pi x}{2l} \sin \frac{i\pi at}{2l}$$

$$\mathbf{5.4-3.} u = \frac{2l^2 Q}{\pi^3 a^2 m} \sum_{i=1}^{\infty} \frac{(-1)^{i-1}}{i^3} \sin \frac{i\pi x}{l} \left( 1 - \cos \frac{i\pi at}{l} \right)$$

$$\mathbf{5.4-4.} u = \frac{P_1 t^4}{12lmt_1^2} + \frac{2lP_1}{\pi^2 a^2 mt_1^2} \sum_{i=2, 4, 6, \dots}^{\infty} \frac{(-1)^{i/2}}{i^2} \cos \frac{i\pi x}{l} \times \left[ t^2 - \frac{2l^2}{(i\pi a)^2} \left( 1 - \cos \frac{i\pi at}{l} \right) \right]$$

### 习题 5.6

$$\mathbf{5.6-1.} u = \left[ 1 + \frac{16l^2 \Omega^2}{\pi^3 a^2} \sum_{i=1, 3, 5, \dots}^{\infty} \frac{\beta_i}{i^3} (-1)^{(i-1)/2} \cos \frac{i\pi x}{2l} \right] d \sin \Omega t$$

$$\mathbf{5.6-2.} u = \frac{u_1}{t_1^2} \left[ t^2 - \frac{8l^2}{\pi^3 a^2} \sum_{i=1, 3, 5, \dots}^{\infty} \frac{1}{i^3} \sin \frac{i\pi x}{l} \left( 1 - \cos \frac{i\pi at}{l} \right) \right]$$

$$\mathbf{5.6-3.} u = \frac{u_1}{t_1^2} \left[ \frac{(l-x)}{l} t^2 - \frac{4l^2}{\pi^3 a^2} \sum_{i=1}^{\infty} \frac{1}{i^3} \sin \frac{i\pi x}{l} \left( 1 - \cos \frac{i\pi at}{l} \right) \right]$$

$$\mathbf{5.6-4.} u = \frac{u_1}{t_2^3} \left[ \frac{x}{l} t^3 + \frac{12l^2}{\pi^3 a^2} \sum_{i=1}^{\infty} \frac{(-1)^i}{i^3} \sin \frac{i\pi x}{l} \left( t - \frac{l}{i\pi a} \sin \frac{i\pi at}{l} \right) \right]$$

**习题 5.10**

**5.10-1.**  $v = \frac{2Pl^3}{\pi^4 EI} \sum_i \frac{(-1)^{(i-1)/2}}{i^4} \sin \frac{i\pi x}{l} \cos \omega_i t \quad (i=1, 3, 5, \dots)$

**5.10-2.**  $v = \frac{2Pl^3}{\pi^4 EI} \sum_i \frac{1}{i^4} \sin \frac{i\pi x}{l} \sin \frac{i\pi x_1}{l} \cos \omega_i t \quad (i=1, 2, 3, \dots)$

**5.10-3.**  $v = \frac{4\omega l^4}{\pi^5 EI} \sum_i \frac{1}{i^5} \sin \frac{i\pi x}{l} \cos \omega_i t \quad (i=1, 3, 5, \dots)$

**5.10-4.**  $v = \frac{4\dot{v}_0}{\pi} \sum_i \frac{1}{i\omega_i} \sin \frac{i\pi x}{l} \sin \omega_i t \quad (i=1, 3, 5, \dots)$

**习题 5.11**

**5.11-1.**  $X_i = \cosh k_i x - \cos k_i x - \alpha_i (\sinh k_i x - \sin k_i x);$

$$\alpha_i = \frac{\cosh k_i l - \cos k_i l}{\sinh k_i l - \sin k_i l}$$

$$k_1 l = 3.927 \quad k_2 l = 7.069 \quad \alpha_1 = 1.0008 \quad \alpha_2 = 1.0000$$

**5.11-2.**  $X_i = \cosh k_i x - \cos k_i x - \alpha_i (\sinh k_i x - \sin k_i x);$

$$\alpha_i = \frac{\cosh k_i l + \cos k_i l}{\sinh k_i l + \sin k_i l}$$

$$k_1 l = 1.875 \quad k_2 l = 4.694 \quad \alpha_1 = 0.7341 \quad \alpha_2 = 1.0185$$

**5.11-3.**  $v = \frac{P_0 l^3}{EI} (1724X_1 \cos \omega_1 t + 380X_2 \cos \omega_2 t + 94X_3 \cos \omega_3 t + \dots) \times 10^{-6}$

**5.11-4.**  $v = \frac{P_0 l^3}{EI} (16182X_1 \cos \omega_1 t - 412X_2 \cos \omega_2 t + 53X_3 \cos \omega_3 t - \dots) \times 10^{-5}$

**习题 5.13**

**5.13-1.**  $(v_m)_{x=l/2} = \frac{2Pl^3}{\pi^4 EI} \left[ \frac{1}{1 - \frac{1}{4}} + \frac{1}{3^4 - \frac{1}{4}} + \frac{1}{5^4 - \frac{1}{4}} + \dots \right] \approx 1.328 \text{ mm}$

**5.13-2.**  $(v_m)_{x=l/2} = \frac{4Pl^3}{\pi^4 EI} \left[ \frac{\sin(\pi/3)}{1 - \frac{1}{4}} + \frac{\sin(5\pi/3)}{5^4 - \frac{1}{4}} + \frac{\sin(7\pi/3)}{7^4 - \frac{1}{4}} + \dots \right] \approx 2.273 \text{ mm}$

**5.13-3.**  $(v_m)_{x=l/2} = \frac{2\omega l^4}{\pi^5 EI} \sum_{i=1, 3, 5, \dots}^{\infty} \frac{(-1)^{(i-1)/2}}{i(i^4 - \alpha^2)}$

**5.13-4.**  $v = \frac{2Pl^3}{\pi^2 EI} \sum_{i=1, 3, 5, \dots}^{\infty} \frac{(-1)^{(i-1)/2}}{i^4} \sin \frac{i\pi x}{l} (1 - \cos \omega_i t)$

**5.13-5.**  $v = \frac{\omega l^4 \beta_1 \sin \Omega t}{\pi^4 EI} \sin \frac{\pi x}{l}; \quad \beta_1 = \frac{1}{1 - \Omega^2/\omega_1^2}$

**5.13-6.**  $v = \frac{2\pi}{ml^2} \sum_{i=1}^{\infty} \frac{1}{P_i} \sin \frac{i\pi x}{l} \int_0^t \cos \frac{i\pi x}{l} \int_0^t M(x, t') \sin \omega_i(t-t') dt' dx$

**习题 5.14**

**5.14-1.**  $v = \frac{Pl^3 \sin \Omega t}{EI} \sum_{i=1}^{\infty} \frac{\beta_i X_i (X_i)_{x=l}}{(k_i l)^4}; \beta_1 = \frac{1}{1 - \Omega^2/\omega_1^2}$

**5.14-2.**  $(v)_{x=l} = \frac{4Pl^3}{EI} (0.08091\beta_1 + 0.00206\beta_2 + 0.00026\beta_3 + \dots) \sin \Omega t$

**5.14-3.**  $v = \frac{Pl^3 \sin \Omega t}{EI} \sum_{i=1}^{\infty} \frac{\beta_i X_i (X_i)_{x=l/2}}{(k_i l)^4}$

**5.14-4.**  $(v)_{x=l/2} = \frac{Pl^3 \sin \Omega t}{EI} (8829\beta_1 + 130\beta_2 + 156\beta_3 + 9\beta_4 + 23\beta_5 + \dots)$

**习题 5.15**

**5.15-1.**  $v = \left[ 1 - \frac{x}{l} + \frac{2l^4 \Omega^2}{\pi^5 a^2} \sum_{i=1}^{\infty} \frac{\beta_i}{i^5} \sin \frac{i\pi x}{l} \right] v_1 \sin \Omega t$

**5.15-2.**  $v = \frac{\theta_2}{t_2^2} \left[ xt^2 - \sum_{i=1}^{\infty} \frac{2X_i}{\omega_i^2} \int_0^t x X_i dx (1 - \cos \omega_i t) \right]$

**5.15-3.**  $u = \left[ -\frac{x^2}{l} + \frac{x^3}{l^2} + \sum_{i=1}^{\infty} \frac{\Omega^2 X_i}{(\omega_i^2 - \Omega^2)} \int_0^l \left( -\frac{x^2}{l} + \frac{x^3}{l^2} \right) X_i dx \right] \theta_4 \sin \Omega t$

**5.15-4.**  $u = \frac{v_3}{t_3^3} \left[ \left( \frac{3x^2}{2l^2} - \frac{x^3}{2l^3} \right) t^3 - \sum_{i=1}^{\infty} \frac{6X_i}{\omega_i^2} \int_0^l \left( \frac{3x^2}{2l^2} - \frac{x^3}{2l^3} \right) X_i dx \left( t - \frac{1}{\omega_i} \sin \omega_i t \right) \right]$

**第 6 章****习题 6.4**

**6.4-1.**  $\mathbf{p}_b(t) = \{2b_1 + b_2, b_1 + 2b_2\} \frac{L}{6}$

**6.4-2.**  $\mathbf{p}_b(t) = \{1, 3\} \frac{b_2 L}{12}$

**6.4-3.**  $\mathbf{K} = \frac{EA}{3L} \begin{bmatrix} 7 & -8 & 1 \\ -8 & 16 & -8 \\ 1 & -8 & 7 \end{bmatrix}$

**6.4-4.**  $\mathbf{M} = \frac{\rho AL}{15} \begin{bmatrix} 2 & 1 & -\frac{1}{2} \\ 1 & 8 & 1 \\ -\frac{1}{2} & 1 & 2 \end{bmatrix}$

**6.4-5.**  $\mathbf{p}_b(t) = \{1, 4, 1\} \frac{b_x L}{6}$

**6.4-6.**  $\mathbf{p}_b(t) = \{5, 3\} \frac{m_{x1} L}{12}$

**6.4-7.**  $\mathbf{p}_b(t) = \{L - x, x\} \frac{M_x}{L}$

**6.4-8.**  $\mathbf{p}_b(t) = \{9, 2L, 21, -3L\} \frac{b_2 L}{60}$

**6.4-9.**  $\mathbf{p}_b(t)_1 = \{2x^3 - 3x^2L + L^3, x^3L - 2x^2L^2 + L^3, -2x^3 + 3x^2L, x^3L - x^2L^2\} \frac{P_y}{L^3}$

$$\mathbf{p}_b(t)_2 = \{6x^2 - 6xL, 3x^2L - 4xL^2 + L^3, -6x^2 + 6xL, 3x^2L - 2xL^2\} \frac{M_z}{L^3}$$

**6.4-10.**  $\mathbf{p}_b(t) = \{21b_1 + 9b_2, (3b_1 + 2b_2)L, 9b_1 + 21b_2, -(2b_1 + 3b_2)L\} \frac{L}{60}$

**6.4-11.** 式(6.75)已给出

**6.4-12.** 式(6.81)已给出

### 习题 6.5

**6.5-1.**  $S_{s11} = \frac{4EI}{L}; S_{s12} = \frac{2EI}{L}; S_{s13} = 0;$  等等

$$M_{s11} = \frac{\rho AL^3}{105}; M_{s12} = \frac{\rho AL^3}{140}; M_{s13} = 0;$$
 等等

**6.5-2.**  $S_{s11} = \frac{8EI}{L}; S_{s12} = \frac{6EI}{L^2}; S_{s13} = \frac{2EI}{L};$  等等

$$M_{s11} = \frac{2\rho AL^3}{105}; M_{s12} = \frac{13\rho AL^2}{420}; M_{s13} = -\frac{\rho AL^3}{140};$$
 等等

**6.5-3.**  $S_{s11} = \frac{12EI}{L^3}; S_{s12} = \frac{6EI}{L^2}; S_{s13} = \frac{6EI}{L^2};$  等等

$$M_{s11} = \frac{13\rho AL}{35}; M_{s12} = \frac{11\rho AL^2}{210}; M_{s13} = -\frac{13\rho AL^3}{420};$$
 等等

**6.5-4.**  $S_{s11} = \frac{8EI}{L^3}; S_{s12} = \frac{6EI}{L^2}; S_{s13} = \frac{2EI}{L};$  等等

$$M_{s11} = \frac{2\rho AL^3}{105}; M_{s12} = \frac{13\rho AL^2}{420}; M_{s13} = -\frac{\rho AL^3}{140};$$
 等等

**6.5-5.**  $\omega_{1,2} = \frac{5.546, 21.04}{L^2} \sqrt{\frac{EI}{\rho A}}; \Phi_1 = \begin{bmatrix} L \\ -0.394 \end{bmatrix}; \Phi_2 = \begin{bmatrix} L \\ -26.61 \end{bmatrix}$

**6.5-6.**  $\omega_{1,2} = \frac{15.14, 28.98}{L^2} \sqrt{\frac{EI}{\rho A}}; \Phi_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}; \Phi_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

**6.5-7.**  $\omega_{1,2} = \frac{13.32, 34.79}{L^2} \sqrt{\frac{EI}{\rho A}}; \Phi_1 = \begin{bmatrix} 1 \\ -0.707 \end{bmatrix}; \Phi_2 = \begin{bmatrix} 1 \\ 0.707 \end{bmatrix}$

**6.5-8.**  $\omega_1 = \frac{15.69}{L^2} \sqrt{\frac{EI}{\rho A}}$

**6.5-9.**  $\omega_{1,2} = 5.603, 31.19 \frac{1}{L^2} \sqrt{\frac{EI}{\rho A}}; \Phi = \begin{bmatrix} 1.000 & 1.000 \\ 0.5435 & -0.9364 \end{bmatrix}$

**6.5-10.**  $\omega_1 = \frac{9.941}{L^2} \sqrt{\frac{EI}{\rho A}}$

**6.5-11.**  $\omega_{1,2,3} = 0, 0, 22.47 \frac{1}{L^2} \sqrt{\frac{EI}{\rho A}}; \Phi = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -0.6 \\ 1 & -1 & 1 \end{bmatrix}$

